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the instrument for his own particular case. By reference to Leslie and Hutton, additional information as to these "aids to calculation" will be received.

I shall only here refer further to the hints thrown out by Professor De Morgan in his evidence as to the extended use of logarithms and of the sliding scales now used in some branches of business. The following is taken from his evidence :—

> "The arbitration of exchanges is a very complicated consideration with the present system. In 1802 a book was published by a gentleman of the name of Teschemacher, for the arbitration of exchanges by the use of logarithms. It did not mention the word 'logarithms,' because that would have frightened the mercantile men. It did not come into use, because it was not properly introduced, and was sold at a very high price; and I only mention it as showing my own knowledge that a system of logarithms would be a very material aid in the question of the arbitration of exchanges. I will mention another instance, and that is the sliding rule which the carpenters and engineers use, which is the logarithmic rule, very easy to understand, and easy to apply to any decimal system. It is not impossible to apply a sliding rule of calculation to our system as it is, but though it is not impossible, it is practically impossible; it is impracticable, as the difficulty of using it with pounds, shillings, and pence, would be too great. Brokers, for instance, and persons who have to make calculations and adaptations very quickly to find out the proper price of one stock according to the price of another stock, might use the sliding rule to very great advantage."

If we adopt a decimal system of numeration and coinage, I have no doubt business men will soon become adepts in the use of all these important aids to calculation, and logarithmic tables will come to be used with as much facility as an ordinary interest table.

On Multiplication by aid of a Table of Single Entry.

By J. J. SYLVESTER, M.A., F.R.S.

A REMARK has been conveyed to me by Mr. Cayley, as occurring somewhere in Gergonne's *Annales*, which is, I think, of sufficient interest for the purposes of practical computation, and especially as applicable to questions of the class which occur in actuarial practice, to justify its introduction to the notice of the readers of this *Journal*. This remark is to the effect that a table of single entry, sufficiently extensive, will serve to give the product of any two numbers by the aid of the processes of addition and subtraction alone, just as is the case in logarithmic computation, but with the advantage over that method of perfect precision in the result, and of the number of findings required for each computation being only two in number, the third and by far the most troublesome of the three findings—viz., that of finding back the number corresponding to the result of the linear (meaning thereby additive or subtractive) process—which occurs in the logarithmic, not arising in the method about to be explained, which we shall presently see admits of being extended, without much additional complexity, to the direct multiplication of three or any greater number of numerical factors.

For two factors a table of squares must be supposed to be given. Such exists in Barlow's tables (which, for the sake of the column of reciprocals which they contain, are in the hands of every actuary, so called, capable of performing his own computations) up to the base 10,000, and in Hutton's tables as far as the base 25,400.

Suppose, now, that a and b are any two numbers both even or both odd, we may proceed as follows :—

Take the two numbers which are the respective half sum and half difference of a and b; find the squares corresponding to these respectively from the given table; the difference of these squares will be the product of the given numbers a and b, as is apparent from the identity,

$$ab=\left(\frac{a+b}{2}\right)^2-\left(\frac{a-b}{2}\right)^2.\text{*}$$

If one of the given numbers is even and the other odd, we may diminish either one of the two given numbers by a unit (it will practically be best to diminish the greater), apply the rule as above stated to the given couple of numbers thus modified, and increase the result by the remaining one of the given numbers (*i. e.*, the lesser of the two, if the greater be the one diminished). We may thus, by aid of Hutton's Table of Squares, multiply any two numbers together whose sum does not exceed 50,801; and, by the more easily accessible tables of Barlow, any two numbers whose sum does not exceed 20,001.

Again, these same tables of Barlow (which contain the cubes of numbers in addition to the squares) give the means of finding the continued product of three numbers by an analogous linear method.

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[*] To render the process given by this formula strictly linear, the table employed should be that of the fourth parts of the squares of numbers (rejecting the fraction for the odd numbers) in place of a table of the squares themselves. A like remark will apply, subject to certain modifications as regards the fractional part, to the general case. For the product of two numbers the scheme of the operations will thus come to consist of an addition, two subtractions, and two direct findings—in lieu of the addition, two direct and one reverse findings, required in the logarithmic process.

For this purpose we must use the formula of identity,

$$abc=\frac{1}{24}\left\{(a+b+c)^3-(a+b-c)^3-(c+a-b)^3-(b+c-a)^3\right\}.$$

If a, b, c are all or only one of them even, this may be put under the form

$$abc=\frac{1}{3}\left\{\left(\frac{a+b+c}{2}\right)^3-\left(\frac{a+b-c}{2}\right)^3-\left(\frac{c+a-b}{2}\right)^3-\left(\frac{b+c-a}{2}\right)^3\right\}^3;$$

and the method may be applied, by aid of Barlow's tables, to any three such numbers (subject to this condition) whose united sum does not exceed 20,000. Practical modifications to meet the contrary supposition will readily suggest themselves to the skilled computer.

It may be worth while to subjoin the general formula for expressing the product of any number (r) of quantities in terms of the rth powers of their linear combinations.*

Let a_1, a_2, a_3 . . . a_r, be the r numbers in question, and in general let θ_1, θ_2, θ_3 . . . θ_r, be disjunctively equal, in some unascertained order of relation, to 1, 2, 3 . . . r; then

$$a_1,\ a_2,\ a_3 \ldots . a_r=\frac{1}{4.6\ldots(2r)}\Big\{\left(a_{\theta_1}+a_{\theta_2}+a_{\theta_3}+\ldots a_{\theta_r}\right)^r-$$

$$\sum\left(-a_{\theta_1}+a_{\theta_2}+a_{\theta_3}+\ldots+a_{\theta_r}\right)^r+\sum\left(-a_{\theta_1}-a_{\theta_2}+a_{\theta_3}+\ldots+a_{\theta_r}\right)^r+,\&c.$$

$$+(-)^i\left(\frac{1}{2}\right)^q\sum\left(-a_{\theta_1}-a_{\theta_2}\ldots-a_{\theta_i}+a_{\theta_{i+1}}\ldots+a_{\theta_r}\right)^r\Big\};$$

where $i=\frac{r}{2}$ and $q=1$ if r is even, and $i=\frac{r-1}{2}$ and $q=o$ if r is odd.

I shall be glad to think that this brief and simple exposition may prove useful in any degree to the practical computer; any saving of labour of this kind, however slight in itself, when multiplied by the number of arithmeticians continuing their labours through countless ages, gives rise to an accumulation of savings which an ordinary feeling of benevolence must show the importance of not disregarding. It is proverbial, that more fortunes are saved than gained; and it is equally true, that the greatest benefits to society result from minute but incessant contributions from a thousand different sources to that general fund of experience, skill, and knowledge, which constitutes by far the most valuable portion of its capital stock.

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* For a proof of this theorem, see *London and Edinburgh Philosophical Magazine*, April, 1854. The total number of terms in the right-hand side of the equation is easily seen to be 2^{r-1}.